most of these errors himself or find the correct letterings in the corresponding diagrams in Heath's translation of the *Elements*. The only error of this kind which caused me some moments of reflection is Eq. (1) on page 100, whose right-hand side should read $\{OA^2 + AD^2\} : AD^2 = \{(CO + OA)^2 + CA^2\} : CA^2$ instead of the concoction $\{(CO + OA)^2 + AD^2\} : AD^2$.

A second technical problem also has to do with the diagrams. Very often these are drawn grossly out of proportion. In a few cases this may have been done for pedagogical reasons, but often the distortions are downright misleading. It required all the reviewer's concentration to conceptualize 11 mm as the half of 28.4 mm or to see a line divided in the ratio 1 : 2.6 as being "really" divided in extreme and mean ratio (1 : 1.61 . . .)—to name but two examples, both to be found on page 30 (Figs. I-25 and I-26). Computers may make nice drawings, but they seem still to be in need of some supervision.

These minor deficiencies are balanced by major merits which the author has achieved in intentional reaction to the "obscure bibliographical references; as well as incorrect translations, incorrect inferences from quotations, and misrepresentation of the mathematical process actually involved in the original" that abound in the literature on this no less than other subjects (p. xi). First, Herz-Fischler's argument is always clear, and clearly arranged. Second, the book as a whole is well organized. Third, and finally, through his worldwide hunt for information on DEMR and for secondary literature touching on the subject the author has accumulated a veritable profusion of references. The contents of the bibliography will be useful to every scholar working in the vicinity of DEMR, and the fullness of the information given will be appreciated by every interlibrary service.

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Die Krise der Anschauung (Studien zur Wissenschafts-, Sozial- und Bildungsgeschichte der Mathematik, 3). By Klaus Th. Volkert. Göttingen (Vandenhoeck & Ruprecht). 1986. xxxii + 420 pp. 98 DM.

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The present volume—an improved version of a doctoral thesis at the University of Saarbrücken—examines the impact of the increasingly abstract development of mathematics on the foundations of the basic concepts. It was motivated by Hans

Hahn's *Wiener Kreis* lecture (1933) bearing the same title. Commenting on the latter's logicist program, the author declares the "hypothesis of the 'crisis of Anschauung' [intuition]" to be "misleading from a systematic point of view, and historically false" (p. 260). Rather, he claims there has been no failure of spatial *Anschauung*, and that mathematical certainty continues to be based on it (p. XVII).

Of the volume's three chapters, the first examines how the role of *Anschauung* has changed from antiquity to modern times from a "historical" point of view. In the second, "historico-systematic" chapter, the concept of *Anschauung* is studied, primarily as used by Kant, and subsequently by mathematicians (such as E. Borel and F. Klein) *after* the alleged crisis. Finally, in the third, "systematic" part, the author develops his own conception of *Anschauung*, contrasting it with formalism, logicism, and intuitionism.

The study does not pertain to the history of mathematics but rather to the philosophy of mathematics. Its historical part gives a summary of familiar secondary literature in order to provide material for philosophical interpretation. There is little use of (published) primary sources. The only detailed presentation concerns the 19th-century program of arithmetization, in particular the Weierstraß example of a continuous, nowhere differentiable function—according to the author's view the classical paradigm of a "monster" that supposedly introduced a crisis of *Anschauung* for the function concept. As the author presents the history of mathematics as a unilinear development that led straight to modern formal theory in a teleological fashion (". . . definitely sealed," "it was necessary"), he is unable to obtain new insights from the clash of opposing views. Worse still, the mathematicians quoted are squeezed ever more tightly into the author's own conceptional pattern, and "standardized" in accordance with his terminology as the book progresses ("Klein overlooks from the very outset that *Anschauung* occurs as a sign activity [*Zeichenhandlung*] in mathematics," p. 241; "Reidemeister must be corrected with respect to . . . ," p. 334, etc.). While many statements are supported by abundant quotations, the author repeatedly offers no evidence at all for crucial interpretations (e.g., p. 231 above).

The author designates his own conception of *Anschauung* as a "pragmatic constructivist" one (p. XXVI) which reestablishes a relationship between the formal and the informal level in mathematics. Volkert is committed to the views of the "Erlangen School" (P. Lorenzen, A. Kamlah). The study's merit is that it presents fundamental problems of the arithmetization program from this viewpoint. The author, however, also confesses that he sees no possibility of saving "direct intuition" as an instance of certainty for mathematics. As "perceived signs" constitute a "residue of intuition" (*Anschauungsrest*, p. 321), the intuition of signs (*Zeichenanschauung*) must be used to found a new certainty. It cannot be overlooked that the proposed method of constituting the respective object in mathematics via "activities of sign production" (p. 383) subsumes the "monsters" in an ever more complex concept of *Anschauung*.

The author makes no allowance for the extensive work of Nelson's school of thought in the philosophy of mathematics which made considerable contributions to the foundations of mathematics on the basis of Kant and Fries after 1900.

A general objection to the volume is that a reinterpretation of foundational conceptions in the philosophy of mathematics cannot be confined to published texts, but must seek out the sources themselves. The traditional presentations are already based on certain selections and judgements. Independent evaluation and the search for new sources would appear to be a more productive approach for the philosophy of mathematics as well.

The History of Mathematics in Finland 1828–1918. By Gustav Elfving. The History of Learning and Science in Finland 1828–1918, Vol 4. Helsingfors (Societas Scientiarum Fennica). 1981. 195 pp.

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The author, Gustav Elfving, was born in 1908 in Helsingfors and died in 1984. Elfving defended his doctoral thesis in 1934, and was a university lector at the Swedish university Åbo Akademi in Åbo (Finland) from 1933 to 1938 before assuming the Swedish Chair of Mathematics at the Helsingfors University. This valuable book, the result of Elfving's painstaking research on the history of mathematics in Finland, is the last to come from his hand. It is part of an ongoing series on the history of science in Finland during the period 1828–1918, which coincides approximately with the period of autonomy under the Tsar of Russia, beginning with Finland's separation after many centuries as a part of Sweden to its full independence in 1917.

In these 90 years an amazing evolution in mathematics took place in Finland, represented by names like Mittag-Leffler, Mellin, the Neovius-Nevanlinna family (Finland's answer to the Bernoullis), and Ernst Lindelöf. Before this period, only one mathematician (primarily an astronomer) who hailed from the province of Finland is worthy of mention: Anders Lexell, a native of Åbo, who succeeded Euler as a member of the Imperial Academy of Sciences in St. Petersburg.

For most of us, it is hard to comprehend the isolation that a pioneer in Finnish mathematics had to overcome in those days. The first professor of mathematics in Helsingfors, Nathanaël af Schultén (whose work spanned the years 1826 to 1855), left Åbo Akademi to make his obligatory study trip to Paris; the travelling time was 33 days! His interests lay in irrational and transcendental numbers, and in continued fractions. At that time he was the only professor of mathematics in Finland. The subsequent upswing in mathematics in Finland was due to the efforts of five men of considerable distinction: af Schultén himself, L. Lorenz Lindelöf (whose work spanned the period 1857–1874), Gösta Mittag-Leffler (1877–1881), Edvard R. Neovius (1883–1900), and Ernst Lindelöf (1903–1938).